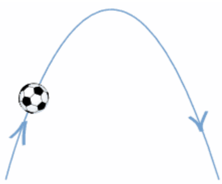
In this activity you will use the equations for motion in a straight line with constant acceleration, and the projectile model to solve problems involving the motion of projectiles.



The problems include finding the time of flight and range of a projectile, as well as finding the velocity and position at a certain time during the motion.

You will need to think about what modelling assumptions are being made and how these assumptions affect the answers.

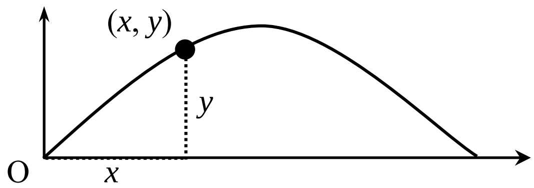
Information sheet

A projectile is a particle that is given an initial velocity, but then moves under the action of its weight alone, that is all other forces are ignored. Real objects such as balls and bullets can be modelled as projectiles.

The motion of a projectile can be studied by splitting it into two components: horizontal motion and vertical motion.

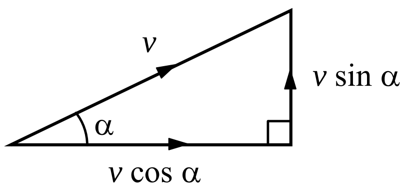
Assume the projectile starts from a point O.

Using horizontal and vertical axes through O allows the position of the projectile at any point in its motion to be given in terms of two coordinates (x, y).



The velocity of the projectile can also be split into two components using a velocity triangle as shown.

When the projectile is travelling with velocity v at an angle α to the horizontal, the horizontal component of its velocity is $v \cos \alpha$ and the vertical component is $v \sin \alpha$.



Think about

How is the right-angled triangle used to find these components?

Assume that the only force on the projectile is its weight. This means that the projectile has a constant acceleration due to gravity, g, vertically ***downwards***, but no horizontal acceleration. If upwards is taken as the positive direction, the acceleration is $-g$ or -9.8 ms^{-2}.

The equations for motion in a straight line with constant acceleration given below can be applied in each direction:

$$v = u + at \qquad s = \frac{(u+v)t}{2} \qquad s = ut + \tfrac{1}{2}at^2 \quad \text{and} \qquad v^2 = u^2 + 2as$$

(where u is initial velocity, v is final velocity, a acceleration, t time taken, and s displacement). The third of these equations gives the following equations for the projectile model.

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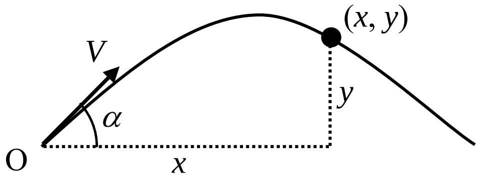
Projectile model

When an object is projected with velocity V at an angle of α to the horizontal and is then assumed to move freely under gravity in a vertical plane, its motion can be modelled by the following equations:



Horizontal motion $x = V\cos\alpha\, t$

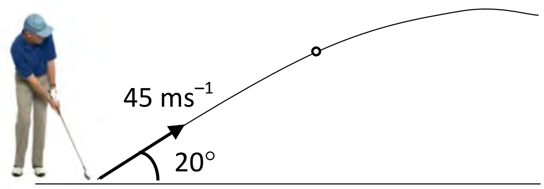
Vertical motion $y = V\sin\alpha\, t - \frac{1}{2}gt^2$

Think about:

How were these equations obtained from $s = ut + \dfrac{1}{2}at^2$?

You can use the equations for horizontal and vertical motion to solve a variety of problems involving projectiles.

Section A Motion of a golf ball



Suppose a golfer hits a ball with a velocity of 45 m s^{-1} at an angle of 20° to the horizontal.

The projectile model can be used to answer some questions about what will happen to the ball later during its flight.

Finding the position at a later time

Where will the ball be 2 seconds later?

Horizontal motion

Using $x = V\cos\alpha\, t$ with V = 45, α = 20° and t = 2

gives x = 45 cos 20° × 2 = 84.57 …. (metres)

Vertical motion

Using $y = V\sin\alpha\, t - \frac{1}{2}gt^2$ with V = 45, α = 20°, g = 9.8 and t = 2

gives $y = 45\sin 20° \times 2 - \frac{1}{2} \times 9.8 \times 2^2$ = 11.18… (metres)

After 2 seconds the ball will be at the point (84.6, 11.2) to 3 sf.

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Finding the highest point

What is the greatest height the ball will reach?

Think about

In what direction will the ball be travelling when it is at its highest point?

When the ball reaches its greatest height its vertical velocity will be zero.
The equation $v = u + at$ can be used to find the time at which this will happen.

Vertical motion

Using $v = u + at$ with $v = 0$, $u = 45 \sin 20°$ and $g = -9.8$

gives $0 = 45 \sin 20° - 9.8t \quad \Rightarrow \quad t = \dfrac{45\sin 20°}{9.8} = 1.5705...$

Now using $y = V\sin\alpha\, t - \frac{1}{2}gt^2$ with $V = 45$, $\alpha = 20°$, $g = 9.8$ and $t = 1.571$

gives $y = 45\sin 20° \times 1.571 - \frac{1}{2}\times 9.8\times 1.571^2 = 12.08...$

The greatest height reached by the ball will be 12.1 metres (to 3 sf)

Finding the time of flight and range

How long will the ball be in the air?

How far will it travel horizontally before it hits the ground?

Assuming that the ball started from ground level and that the ground is horizontal, it will reach the ground again when $y = 0$.

Think about

Why is it important not to confuse y with vertical *distance*?

Vertical motion

Using $y = V\sin\alpha\, t - \frac{1}{2}gt^2$ with $y = 0$, $V = 45$, $\alpha = 20°$ and $g = 9.8$

gives $0 = 45\sin 20°\, t - \frac{1}{2}\times 9.8\times t^2 \quad \Rightarrow \quad 0 = 15.39\,t - 4.9t^2$

Factorising gives $\quad t(15.39 - 4.9\,t) = 0$

$\Rightarrow \quad t = 0$ or $t = \dfrac{15.39}{4.9} = 3.141...$

The ball will be in the air for 3.14 seconds (to 3 sf)

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Think about

Why is $t=0$ also a solution of $0=45\sin 20°\,t-\frac{1}{2}\times 9.8\times t^2$?

How would the problem change if the ground was not horizontal?

Horizontal motion

Using $x=V\cos\alpha\, t$ with V = 45, α = 20° and t = 3.141

gives x = 45 cos 20° × 3.141 = 132.82…

The ball will be 133 metres (to 3sf) from its starting point when it hits the ground.

> Note that sometimes the equation for y gives a quadratic equation that cannot be factorised. In such cases you will need to use the quadratic formula:
>
> The solutions of the equation $at^2+bt+c=0$ are $t=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Finding how long it takes to reach a particular height

How long does it take the golf ball to reach a height of 10 metres?

Vertical motion

Using $y=V\sin\alpha\, t-\frac{1}{2}gt^2$ with y = 10, V = 45, α = 20° and g = 9.8

gives $10=45\sin 20°\,t-\frac{1}{2}\times 9.8\times t^2 \Rightarrow \quad 10=15.39\,t-4.9t^2$

Rearranging this gives $4.9t^2-15.39t+10=0$

Using a = 4.9, b = – 15.39 and c = 10 in the quadratic formula gives

$$t=\frac{15.39\pm\sqrt{15.39^2-4\times 4.9\times 10}}{2\times 4.9}$$ = 2.222… or 0.9182…

Think about

Why are there two answers for t?

The ball is 10 metres above the ground twice – once on the way up and once on the way down. The ball will first reach a height of 10 metres after 0.918 seconds (to 3 sf).

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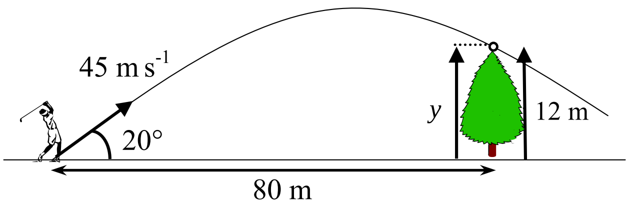
Finding out whether a projectile will clear an obstacle

For example, if the golf ball is hit in the direction of a 12 metre tree which is 80 metres from the golfer, will the ball pass over the tree or hit it?



Horizontal motion

Using $x = V\cos\alpha\, t$ with x = 80, V = 45 and α = 20°

gives 80 = 45 cos 20°t $\Rightarrow$ $t = \dfrac{80}{45\cos 20°}$ = 1.8918…

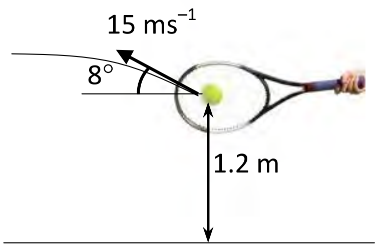
Vertical motion

Using $y = V\sin\alpha\, t - \frac{1}{2}gt^2$ with V = 45, α = 20°, t = 1.892 and g = 9.8

gives $y = 45\sin 20° \times 1.892 - 4.9 \times 1.892^2$ = 11.579…

This is less than 12 metres, so the ball will hit the tree.

Section B Motion of a tennis ball



Sometimes a ball may not start from ground level.

Suppose a tennis player hits a ball when it is at a height of 1.2 metres. giving it a velocity of 15 m s^{-1} at an angle of 8° to the horizontal.

Finding when and where the ball will hit the ground

Vertical motion

Take the origin at the point where the ball was hit.

Using $y = V\sin\alpha\, t - \frac{1}{2}gt^2$ with y = – 1.2, V = 15, α = 8° and g = 9.8

gives $-1.2 = 15\sin 8° \times t - 4.9 \times t^2 \Rightarrow$ $-1.2 = 2.088\,t - 4.9t^2$

Rearranging this gives $4.9t^2 - 2.088t - 1.2 = 0$

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Using a = 4.9, b = – 2.088 and c = – 1.2 in the quadratic formula

gives $t = \dfrac{2.088 \pm \sqrt{2.088^2 + 4 \times 4.9 \times 1.2}}{2 \times 4.9}$ = 0.7518 or – 0.3257

Think about

Why is the second answer for t not valid?

The ball hits the ground after 0.752 seconds (to 3 sf).

Horizontal motion

Using $x = V\cos\alpha\, t$ with V = 15, α = 8° and t = 0.7518

gives x = 15 cos 8° × 0.7518 = 11.167…

The ball hits the ground a horizontal distance of 11.2 metres (to 3 sf) from the point where it was hit by the tennis player.

Try these

Use the projectile model to answer the following questions.

Take g = 9.8 m s^{-2}

At the end of each question state the assumptions you have made.

Also say how you think the results in the real situation might differ from the answers you have given, and whether your model gives an underestimate or an overestimate.

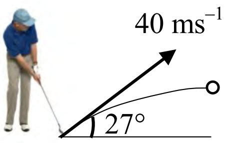
1 A footballer kicks a ball on horizontal ground giving it an initial velocity of 25 m s^{-1} at an angle of 35° to the horizontal.

a Where will the ball be 1.2 seconds after it is kicked?

b What will be the greatest height reached by the ball?

c Where will the ball land?

2 A golfer hits a ball from horizontal ground, giving it an initial velocity of 40 m s^{-1} at an angle of 27° to the horizontal.



a Where will the ball be 2.5 seconds after it is hit?

b What will be the greatest height reached by the ball?

c Where will the ball land?

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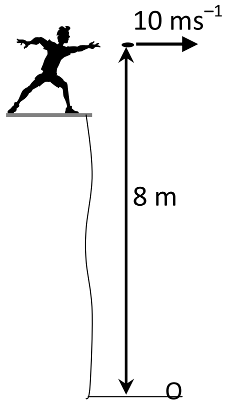
3 A boy throws a stone from the top of a cliff into the sea. When the stone leaves his hand it is travelling horizontally at a velocity of 10 m s^{-1}.

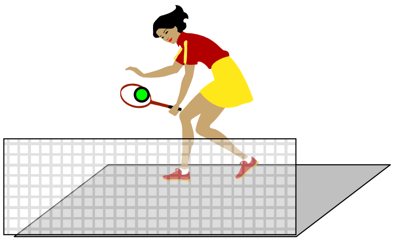


a How long will it take the stone to reach the sea, a distance 8 metres below?

b How far from the bottom of the cliff will it be when it enters the sea?

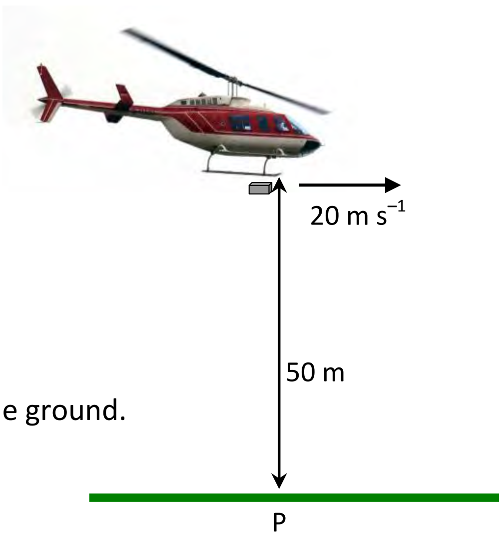
c What will its vertical velocity be when it hits the water?

4 A tennis player hits a ball when it is at a height of 0.6 metres above the court, giving it a velocity of 8 m s^{-1} at an angle of 18° above the horizontal towards the net.



a The net is 0.9 metres high and stands 2 metres from the player. Show that the ball will just pass over the net.

b Find when and where the ball hits the ground.



5 A helicopter is travelling horizontally at 20 m s^{-1} at a height of 50 metres above a point P on horizontal ground when it releases a package.

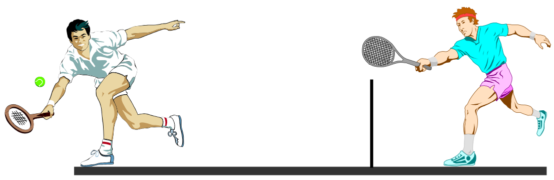
a How long will it take the package to reach the ground?

b How far from P will the package land?

c Calculate the vertical velocity of the package when it reaches the ground.

6 A tennis player hits a ball when it is 0.4 metres above the ground and 3 metres from the net, giving it a velocity of 15 m s^{-1} at 60° to the horizontal.

The tennis player's opponent who has run to the net can reach balls to a height of 2.7 metres. Will he be able to reach this ball?



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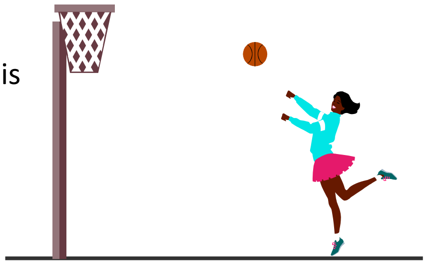
7 A netball player throws a ball at 6.5 m s^{-1} at an angle of 50° to the horizontal towards the net. When the ball leaves her hands it is 1.8 metres from the ground and 3.5 metres horizontally from the nearest part of the net.
The height of the top of the net is 3 metres.

Show that the ball will go under the top of the net.

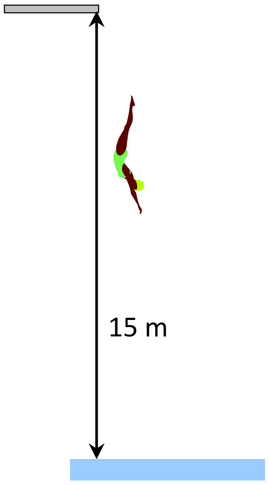


8 At a shooting range, a bullet is fired horizontally from a rifle with a velocity of 800 m s^{-1}.

Find how far it will fall by the time it reaches the target if it is initially

a 100 metres away **b** 200 metres away.

9 A diver steps off a high diving board 15 metres above a pool.



a Sketch a graph showing how the diver's height above the pool's surface varies with time. On your graph show how long it takes the diver to hit the water.

b On her next dive the diver runs along the platform so that she is travelling horizontally at 3 m s^{-1} per second when she leaves it.
i How will the graph of her height above the pool's surface and the time taken to reach the water compare with that of her first dive?
ii How far horizontally, from the point on the pool's surface directly below the edge of the diving board, will the swimmer be when she hits the water?

Extension

Investigate how the velocity of projection affects the motion of a projectile.

What happens to the time of flight and the range if you double the velocity of projection? What happens if the body is projected horizontally?

You could either investigate this for one of the problems you have already solved, or work in general using the projectile model.

Reflect on your work

How does separating the vertical and horizontal motion help you to solve the problems?

What affects the time of flight of a body if it is projected at an angle?

What affects the time of flight of a body if it projected horizontally?

How have your modelling assumptions affected your solutions?

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